

**Year 12 Mathematics Applications**  
**Test 3 2021**  
**Graphs and Networks**  
Section 1 – Calculator Free

**STUDENT'S NAME** \_\_\_\_\_

**DATE:** Thursday 13th May

**TIME:** 25 minutes

**MARKS:** 27

**INSTRUCTIONS:**

Standard Items: Pens, pencils, drawing templates, eraser

Questions or parts of questions worth more than 2 marks require working to be shown to receive full marks.

---

1. (5 marks)

A group of 5 Students wish to organise a ‘paper-scissors-rock’ tournament. Each student must play against every other student (in the group of 5).

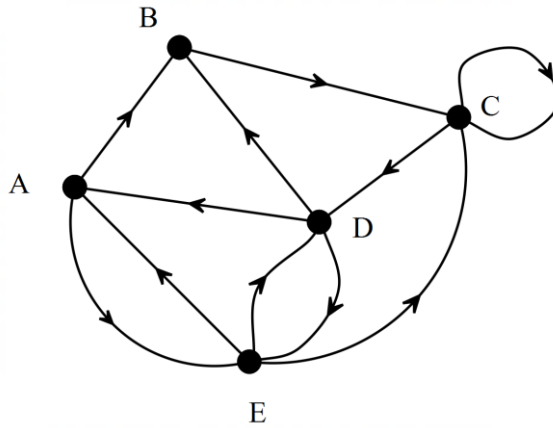
(a) How many matches will need to be played in total? [2]

(b) What is the name of the type of graph will need to be used to show how the matches are organised? [1]

(c) Draw a graph to show how the matches are to be played against each other. [2]

2. (8 marks)

The digraph below shows the walkways between different tourist attractions.



(a)

(ii) Complete the adjacency matrix ' $M$ ' below for the graph. [3]

		To				
		A	B	C	D	E
From	$M =$	A	B	C	D	E
	0	1	0	0	1	
	0	0	1	0	0	
	C					
	D					
E						

(i) What does the sum of the values in row 1 represent? [1]

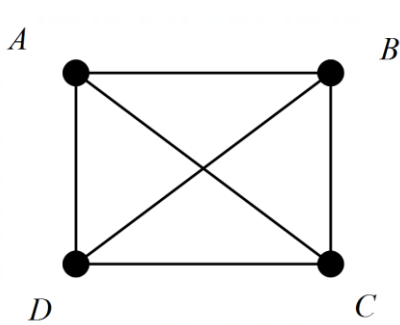
(b) A visitor walks the following route: ABCDECD. Is this route a Hamiltonian path? Justify your answer. [2]

- (c) The matrix  $M^3$  has been constructed below. Explain what the entry in row 4, column 3 tells us. [2]

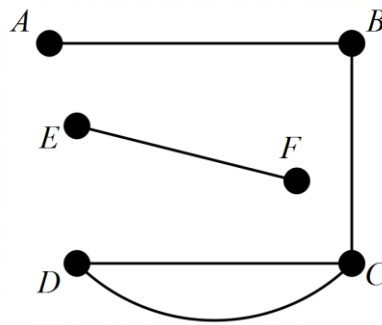
$$M^3 = \begin{matrix} & \begin{matrix} \underline{A} & B & C & D & \underline{E} \end{matrix} \\ \begin{matrix} A \\ B \\ C \\ D \\ E \end{matrix} & \begin{bmatrix} 1 & 2 & 2 & 2 & 2 \\ 1 & 1 & 1 & 1 & 1 \\ 2 & 2 & 3 & 2 & 2 \\ 2 & 2 & 4 & 3 & 2 \\ 3 & 2 & 5 & 3 & 2 \end{bmatrix} \end{matrix}$$

3. (9 marks)

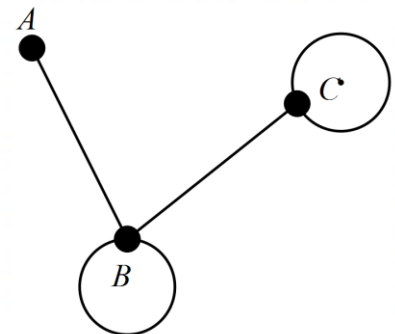
Consider the following graphs



Graph 1



Graph 2

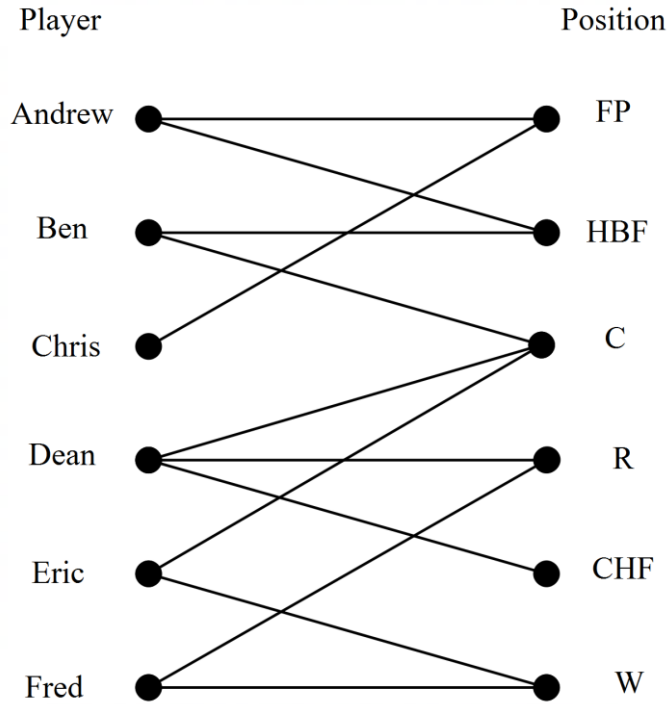


Graph 3

- (a) Which of the above graphs are NOT simple? Justify. [3]
- (b) What is the degree sum of Graph 2? [1]
- (c) State a walk of length 4 for Graph 1 [1]
- (d) Which of the above graphs are connected? [2]
- (e) Verify that Graph 3 satisfies Euler's formula. [2]

4. (5 marks)

Six football players: Andrew, Ben, Chris, Dean, Eric and Fred are trying to decide who should play in which of the following positions: Forward Pocket (FP), Half-Back-Flank (HBF), Wing (W), Centre (C), Rover (R), and Centre-Half-Forward (CHF). The following graph shows which positions each player could possibly fill:



(a) What type of graph has been used above to display the position options? [1]

(b) Which player must play 'Half-Back-Flank'? [1]

(c) Which position must each of the following players play in?

(i) Dean [1]

(ii) Fred [1]

(iii) Ben [1]

**Year 12 Mathematics Applications**  
**Test 3 2021**

Section 2 Calculator Assumed

**STUDENT'S NAME** \_\_\_\_\_

**DATE:** Thursday 13<sup>th</sup> May

**TIME:** 25 minutes

**MARKS:** 24

**INSTRUCTIONS:**

Standard Items: Pens, pencils, drawing templates, eraser

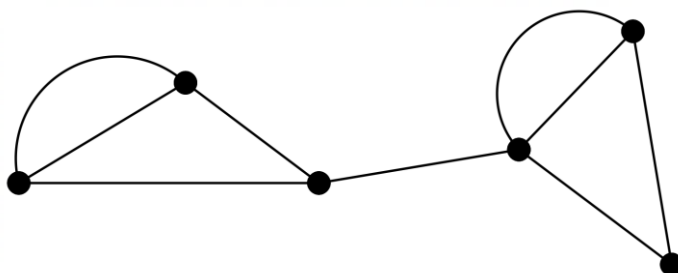
Special Items: Three calculators, notes on one side of a single A4 page (these notes to be handed in with this assessment)

Questions or parts of questions worth more than 2 marks require working to be shown to receive full marks.

---

5. (4 marks)

The graph below shows the walkways connecting exhibits at a Zoo.



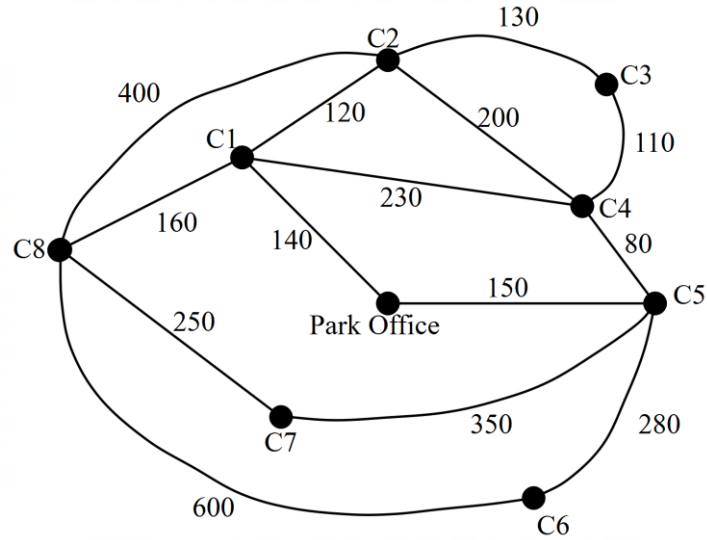
(a) Define the graph as Hamiltonian, Semi-Hamiltonian or Neither. [1]

(b) How is it possible to tell that the graph does not contain an Eulerian Circuit or a Semi-Eulerian Trail? [1]

(b) Draw one edge on the graph so that it will now contain a Semi-Eulerian Trail and will no longer contain a bridge. [2]

6. (11 marks)

The diagram below shows a network of walking tracks between various campsites and the office in a national park. The lengths of the tracks in metres are also shown.



(a) A ranger at C8 plans to visit C1, C3 and C7 before returning to the park office. What is the shortest distance he will have to travel? State the route the ranger must take. [3]

(b) Each day, the ranger on duty has to inspect each of the tracks to ensure they are safe.

(i) Is it possible to do this starting and finishing at the Park Office without repeating a track? Explain why. [2]

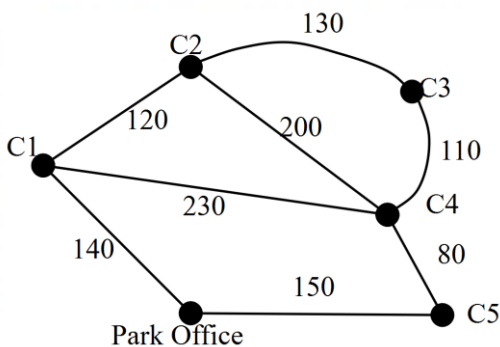
(ii) What is the name for the type of route mentioned in part (b) (i)? [1]

(c) With the present layout of tracks, it is not possible for a ranger to start at the Park Office, inspect each campsite once without repeating a campsite, and return to the Park Office.

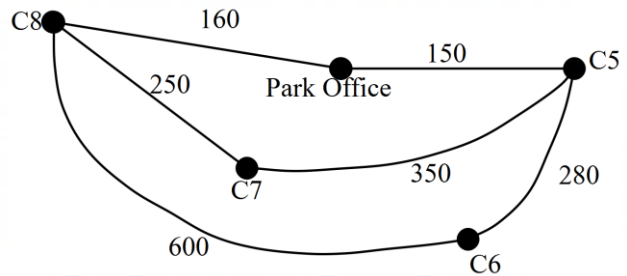
(i) Suggest where an additional track could be added to solve this problem. [1]

(ii) With this new track, write down a route the ranger could follow to start at the Park Office, inspect each campsite exactly once and return to the Park Office. [2]

(d) Rangers decide to break the map up into ‘subgraphs’ to show the campsites that a Ranger is responsible for monitoring during their shift. Which of the following is NOT a correctly constructed subgraph? Explain your answer. [2]



Subgraph 1



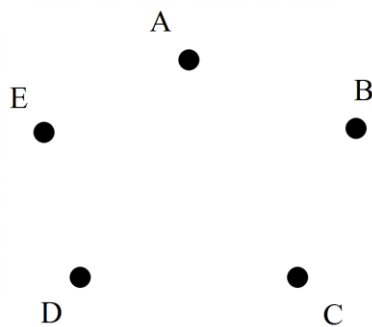
Subgraph 2

7. (9 marks)

(a) Draw a connected graph that has four vertices, four edges, two faces and two bridges. [2]

(b) Determine the number of edges for a planar graph that has five faces, and five vertices. [2]

(c) Draw the graph from part (b) in the space below, clearly showing that it's planar. [2]



(d) State the definition of a Hamiltonian Cycle AND give an example of a Hamiltonian Cycle for your graph in part (c). [2]

(e) A simple connected graph contains  $n$  vertices. Determine the minimum number of edges it contains. [1]